



Computer Studies of Some Effects of Skew
Multipole Components in ES/D Dipoles

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1. INTRODUCTION

Program TEVLAT (1) has been used to investigate the effects of the skew 4- through 30-pole components in the ES/D dipole magnets. Use of correction elements to compensate the skew 4- and 6-pole 0-th harmonics has been studied. With the correction element distributions used, effective cancellation of the 0-th harmonics was easily obtained. Weaker but still noticeable coupling of the betatron oscillations remained after the corrections were used.

Coupling of the vertical and horizontal betatron oscillations by the skew fields induces some new features into the momentum dependence of the tune and therefore also of the other betatron functions. A qualitative explanation of the observed behaviour is suggested by comparison with the coupled harmonic oscillator.

2. MAGNETS

The dipole magnet sets used for this work have been described in an earlier note (2). The means and standard

deviations of the skew 4- through 12-pole components for the three magnet sets are given in Table 1. Table 2 shows the probabilities that a given coefficient falls outside the nominal acceptance criteria given in the Tevatron Design Report (3). As was the case for the normal multipoles, substantial fractions of the skew coefficients are generated outside the nominal limits. Regardless, the coefficients are accepted as generated. The results in this note will therefore tend to overestimate the effects of field errors.

Present plans for skew corrections call for correcting only the 4- and 6-pole terms. For this work the skew correction elements were placed at each normal-cell quadrupole, except at stations 15, 21, 25, 29, 35, 39, and 45. The correction elements are powered in two circuits, one adjacent to the F-quads, the other adjacent to the D-quads. Correction strengths are specified in terms of magnet excitation current, 50 A corresponding to full field (3). The skew element correction currents used are listed in Table 3.

3. RESULTS

Figs. 1 and 2 show the modifications to the tune and beta across the momentum aperture that result when various multipole components from magnet set M1 are added to the perfect linear machine. For these examples, the tunes of the reference machine have been split by 0.015 using the trim quads, and the 0-th harmonic of the 4- and 6-pole components have been compensated by the correction elements. The most notable features of these results are the large effect of the octopole term (Figs. 2a and 2e) and the slight increase in tune splitting due to the skew quadrupole term (Fig. 1b). As indicated in Table 2, the skew octopole term represents a rather severe over-estimate of the probable spread of values for this component. Other results (see below) suggest that this over-estimate does not significantly alter the final results.

Figures 3, 4, and 5 show results obtained using magnet sets M1, M2, and M3 when all normal and skew multipoles and nominal 0-th harmonic corrections are included. These results are directly comparable to the results for normal multipoles only (2).

A striking feature of Figs. 3, 4, and 5 is the apparent discontinuities in the tune versus dp curves and the corresponding jumps in alpha, beta, and gamma. In fact, these jumps are artifacts of the way we have chosen to represent the

data and do not represent real discontinuous changes in the motion. The tune versus dp curves can easily and naturally be drawn as smooth curves representing the tunes of the normal modes of the coupled oscillations. However, for accelerator diagnostics, one likes to think in terms of "x-like" and "y-like" tunes, these being identified as the tunes most sensitive to small changes in the strengths of the F- and D- circuit trim quads, respectively. When this interpretation is used, the jumps are a natural consequence. This problem is discussed more fully in the next section.

Fig. 6 is based on a magnet sample generated so that each multipole component has less than 5% probability of being outside the nominal acceptance limits (3). This example shows that the general features of the results are not significantly dependent on the broad spreads used for the normal decapole and skew octopole components in Figs. 3 - 5. However, the tune jump near $dp/p=0.001$ is not seen in this data (Fig. 6a) and the momentum dependence of the momentum dispersion function is much weaker.

4. DISCUSSION

Following Courant and Snyder (4), coupled betatron oscillations can be described by a 4×4 transfer matrix. The tunes of the normal modes for the betatron oscillations are given by the real parts of the complex eigenvalues of the transfer matrix. (For stable oscillations the eigenvalues are complex conjugate pairs lying on the unit circle.) In the absence of x-y coupling, the 4×4 transfer matrix reduces to block diagonal form, giving independent 2×2 transfer matrices for the x and y oscillations. In this case, the tunes are equivalently given by the traces of the 2×2 submatrices.

When only normal error fields are included in the dipoles, the x and y betatron oscillations are not coupled. Then the x-tune is most sensitive to variations in the strength of the F-circuit trim quads, and the y-tune, to the D-circuit quads. When skew multipoles are included it is found that for most momenta this separation persists. Effectively, the normal modes of the coupled motion are predominantly "x-like" and "y-like". For certain momenta, however, this distinction between the modes vanishes. Furthermore, above and below such a point the the "x-like" and "y-like" behaviors are associated with opposite branches of the tune versus momentum curves. It is this behavior that leads to the tune jumps shown in Figs. 3 - 6.

In TEVLAT, a naive extension of the linear theory described

by Courant and Snyder is used to define approximations to the usual synchrotron functions when non-linearities and/or coupling fields are included. Specifically, the appropriate elements of the on-diagonal 2x2 submatrices and values of $\cos(\mu)$ and $\sin(\mu)$ obtained from the eigenvalues are used in the conventional definitions of alpha, beta, and gamma. This approximation has been found to be consistent so long as the error fields are small and/or compensated by the correction elements. The approximation deteriorates near resonances or when the normal modes do not have a well-defined "x-like" or "y-like" character. In the data shown, the behaviour of the synchrotron functions near the tune jumps should be used as a qualitative guide only.

The origin of this behavior may be seen by considering a simple coupled oscillator system described by

$$\frac{d^2x}{dt^2} + \omega_x^2 x = Cy$$

$$\frac{d^2y}{dt^2} + \omega_y^2 y = Cx.$$

Assuming the normal mode solutions

$$x = ae^{i\omega t}$$

$$y = be^{i\omega t}$$

we find for the frequencies of the normal modes

$$\omega_{\pm}^2 = \frac{1}{2} \left[\omega_x^2 + \omega_y^2 \pm \sqrt{(\omega_x^2 - \omega_y^2)^2 + 4C^2} \right]$$

and their relative amplitudes are

$$\frac{a}{b} = \frac{C}{\omega_x^2 - \omega_{\pm}^2}$$

Now if the frequencies of the uncoupled oscillators are allowed to vary as

$$\omega_x = \omega_0 + u$$

$$\omega_y = \omega_0 - u$$

then ω_{\pm} and a/b will also vary with u . Results for a typical case are sketched in Fig. 7. If we label the motion as "x-like" when $|a/b| > 1$, then the frequency of the "x-like" mode must jump from one branch to the other of the dispersion curves at $u=0$.

For the betatron oscillations, it is natural to think of the traces of the diagonal 2×2 submatrices as defining the "unperturbed tunes", corresponding to the resonant frequencies of the uncoupled oscillators. Fig. 8 shows the values of the traces of the submatrices together with the corresponding eigenvalues for a typical case. The behavior seen here is identical to that of the coupled oscillator example.

I want to thank Don Edwards for valuable suggestions concerning the contents of this note.

REFERENCES

1. A. D. Russell, UPC-124, March, 1980
2. A. D. Russell, UPC-141, Oct., 1980
3. Tevatron Design Report, Fermilab, May, 1979
4. E. D. Courant, H. S. Snyder, Ann. Phys., 3, 1 (1958)

	M1	M2	M3
4-pole	0.267 <u>+1.635</u>	0.207 <u>+1.629</u>	0.245 <u>+1.570</u>
6-pole	-0.267 <u>+1.058</u>	-0.291 <u>+1.055</u>	-0.271 <u>+1.019</u>
8-pole	-0.661 <u>+2.361</u>	-0.489 <u>+2.442</u>	-0.543 <u>+2.354</u>
10-pole	-0.110 <u>+0.519</u>	-0.116 <u>+0.519</u>	-0.118 <u>+0.520</u>
12-pole	-0.317 <u>+0.831</u>	-0.250 <u>+0.844</u>	-0.250 <u>+0.837</u>

Table 1. Means and standard deviations of the randomly distributed skew multipole coefficients for the three sets of 774 dipoles. Units of 10^{**-4} in**-n.

	lower limit	upper limit
4-pole	0.0441	0.0812
6-pole	0.0537	0.0172
8-pole	0.2822	0.1398
10-pole	0.0001	0.0002
12-pole	(0.0220)	(0.0042)

Table 2. Probabilities that the randomly distributed multipole coefficients are outside the ranges defined by the nominal selection criteria given in the Tevatron Design Report. The 12-pole estimates are based on assumed limits of +2 units.

	M1	M2	M3
4-pole	-1.085	-0.841	-0.995
	-1.404	-1.088	-1.288
6-pole	1.035	1.125	1.049
	1.737	1.887	1.760

Table 3. Estimated skew correction currents for the 3 magnet sets. The values given are for the trim elements adjacent to the F and D quads respectively. Full field corresponds to 50 A.

FIGURE CAPTIONS

- Fig. 1. Tune and beta for linear reference, corrected skew 4-pole, corrected skew 6-pole, and corrected 4- plus 6-pole configurations. Magnet set M1.
- Fig. 2. Tune and beta for skew 8-, 10-, 12-, and 14- through 30-pole added to linear machine. Magnet set M1.
- Fig. 3. Tune, betatron functions alpha, beta, and gamma, and momentum dispersion functions. Magnet set M1. All dipole error field components and corrections included.
- Fig. 4. Tune, betatron functions alpha, beta, and gamma, and momentum dispersion functions. Magnet set M2. All dipole error field components and corrections included.
- Fig. 5. Tune, betatron functions alpha, beta, and gamma, and momentum dispersion functions. Magnet set M3. All dipole error field components and corrections included.
- Fig. 6. Tune, betatron functions alpha, beta, and gamma, and momentum dispersion functions. Magnet set chosen so that each multipole component has 5% probability of being outside nominal acceptance limits as given in the Tevatron Design Report. All error field components and corrections included.
- Fig. 7. Coupled harmonic oscillator illustration.
- Fig. 8. Detail of traces of 2x2 submatrices and corresponding $\cos(\mu)$ values near a tune jump.

Axis scales:

Horizontal - $\delta p/p$, -0.5% to 0.5%
Vertical - alpha, -1.0 to 1.0
- beta, 0 to 5000 inches
- gamma, 0 to 8×10^{-4}
- eta, -15 to 110 inches
- eta-prime, -0.02 to 0.03
- tune, 19.20 to 19.60

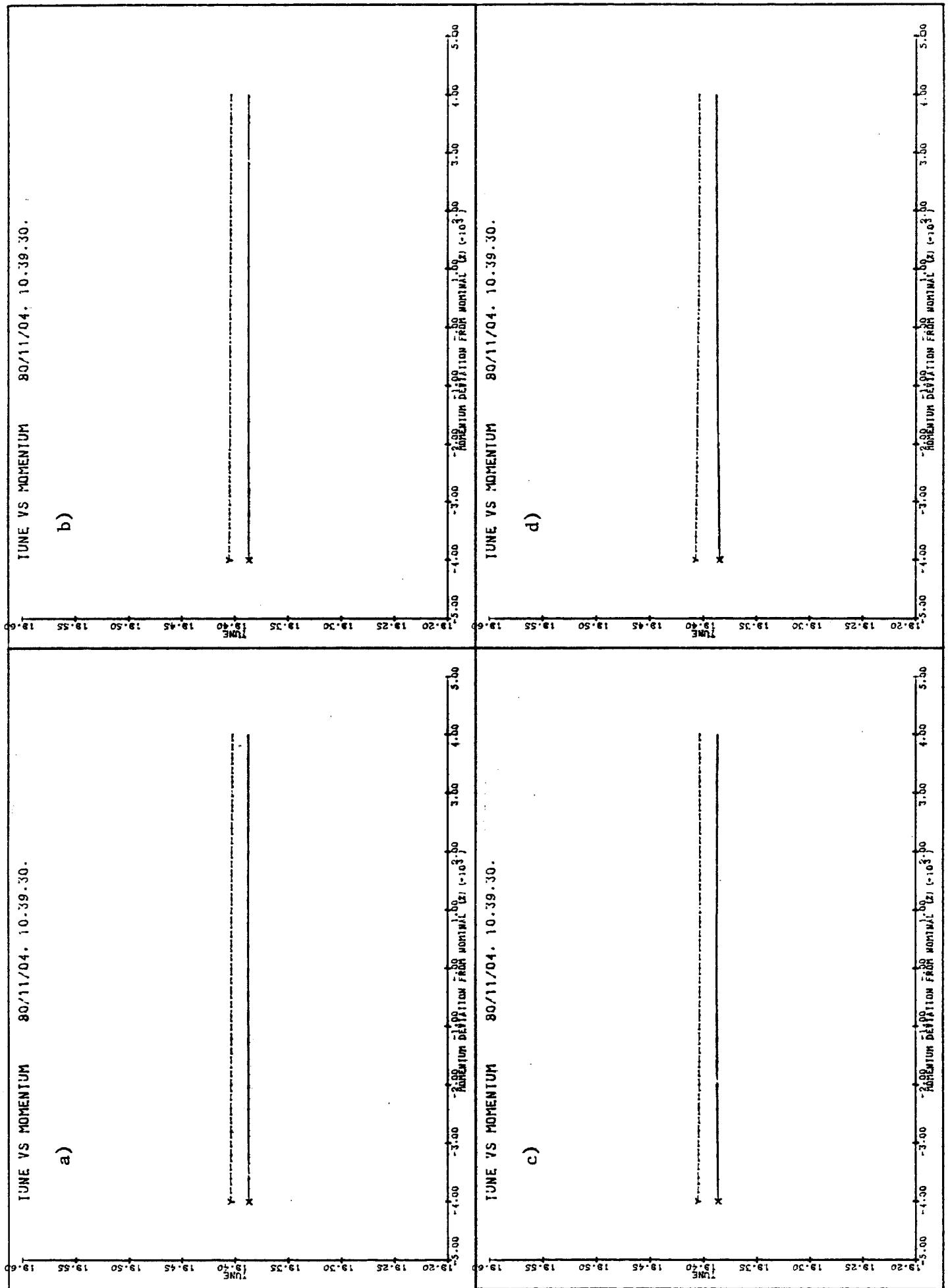


Figure 1

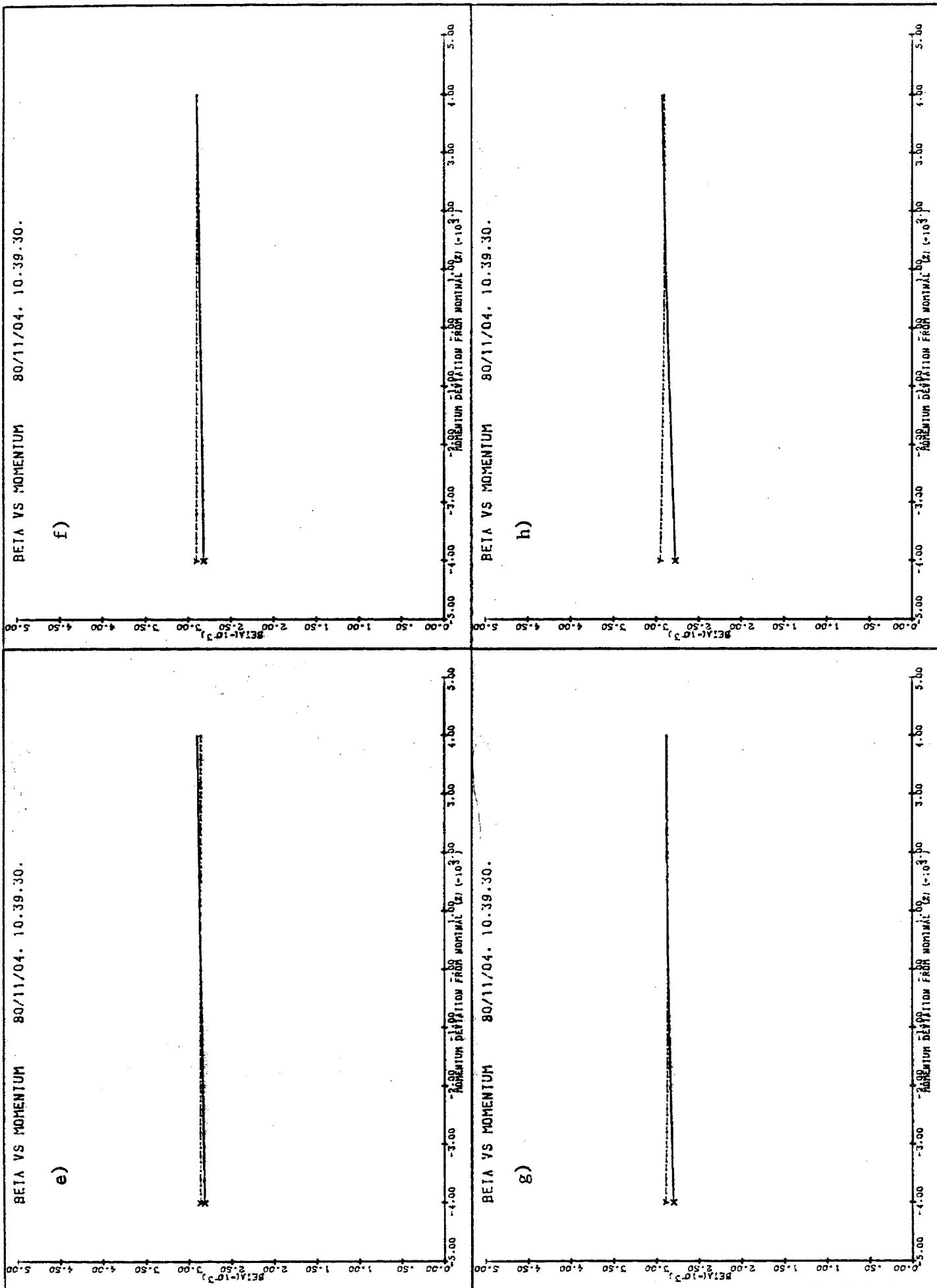


Figure 1

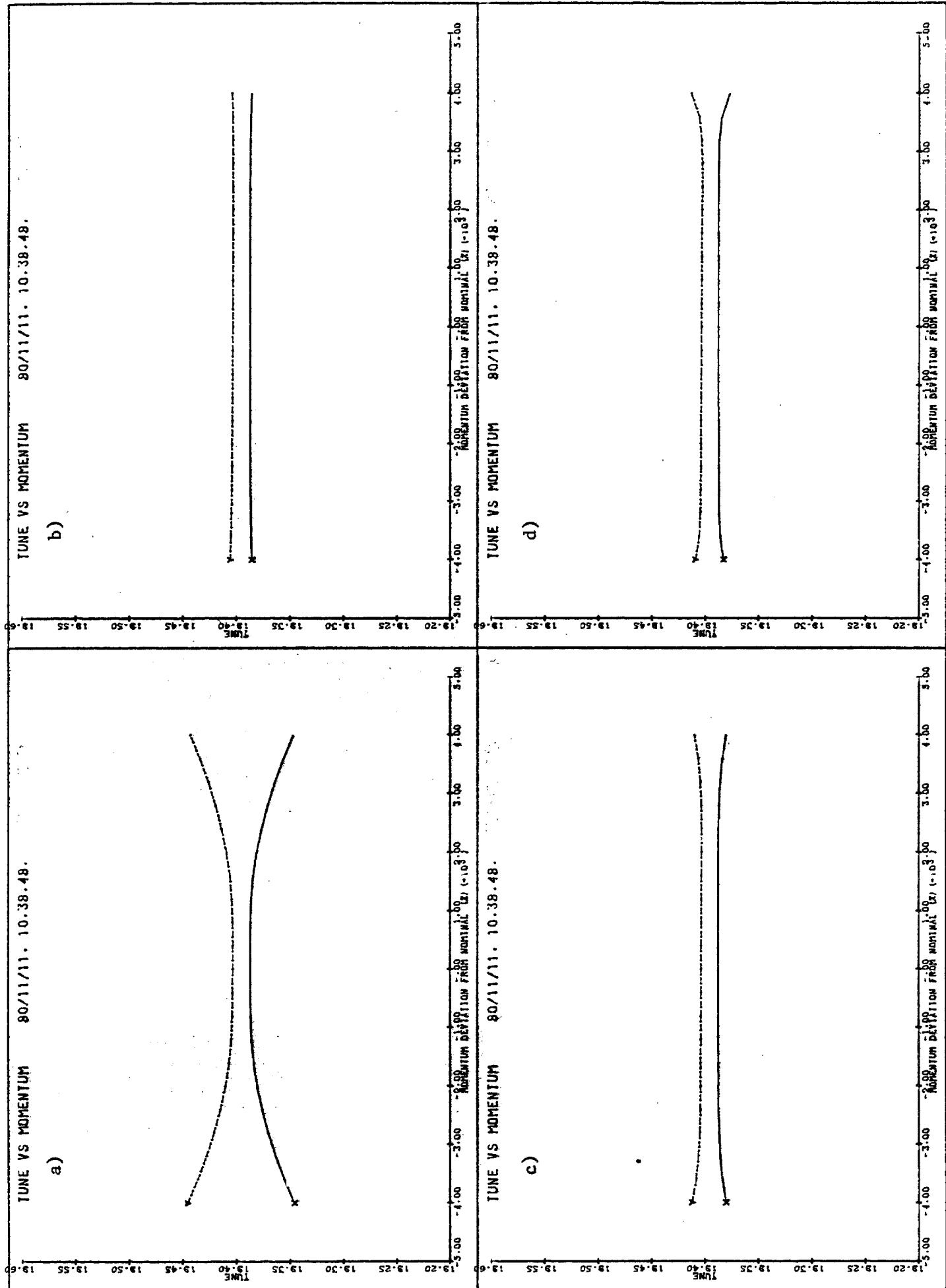


Figure 2

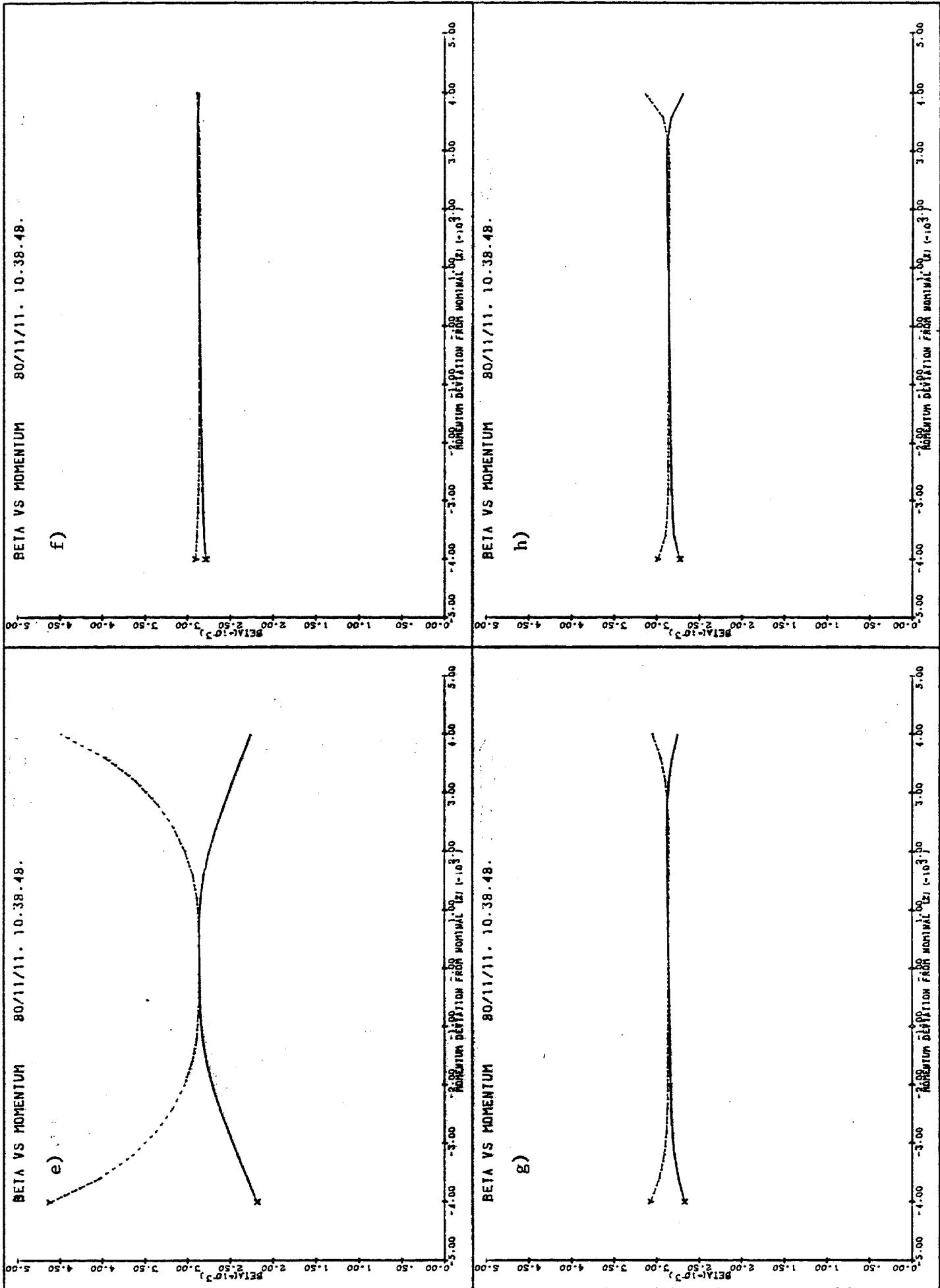


Figure 2

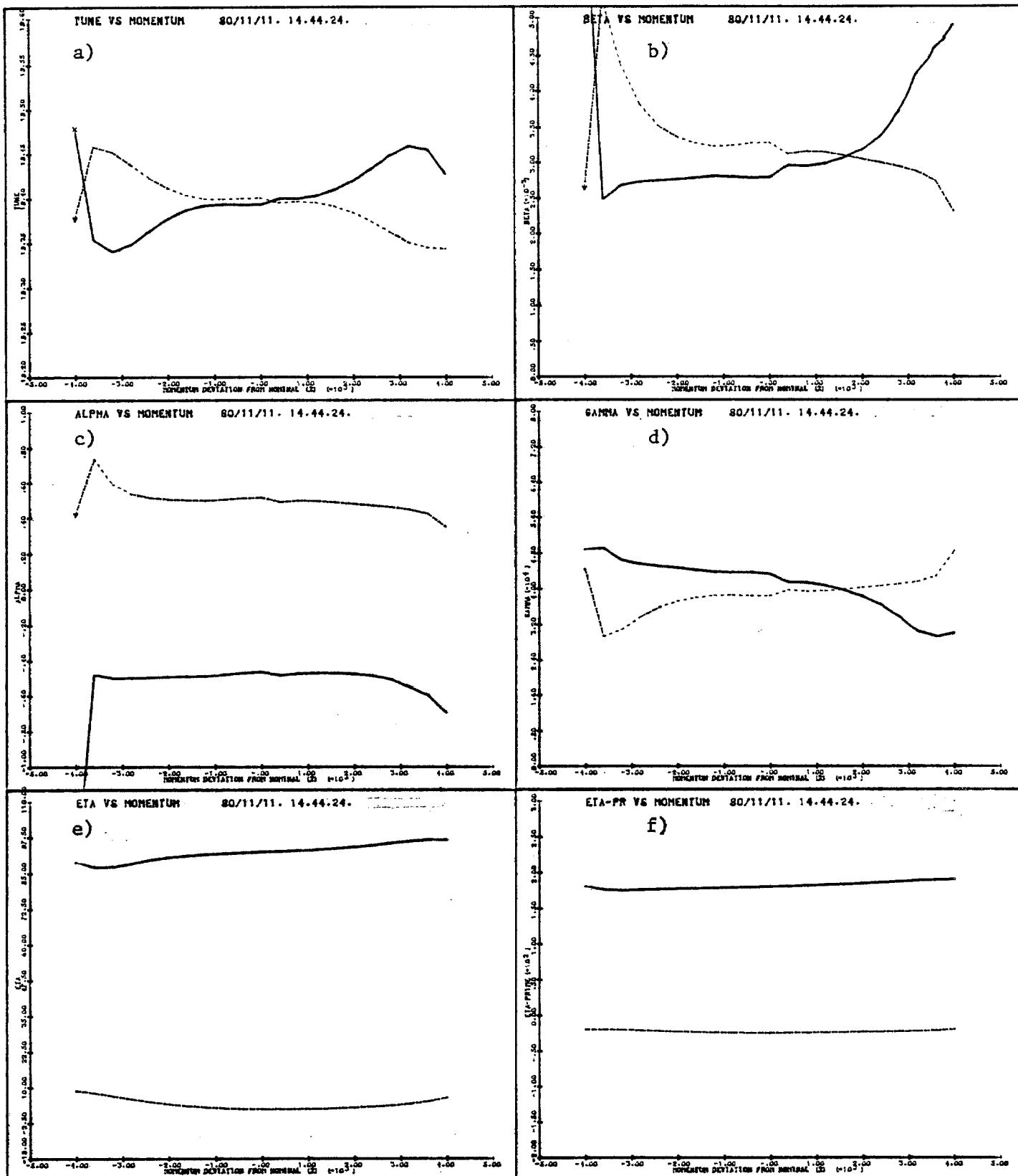


Figure 3

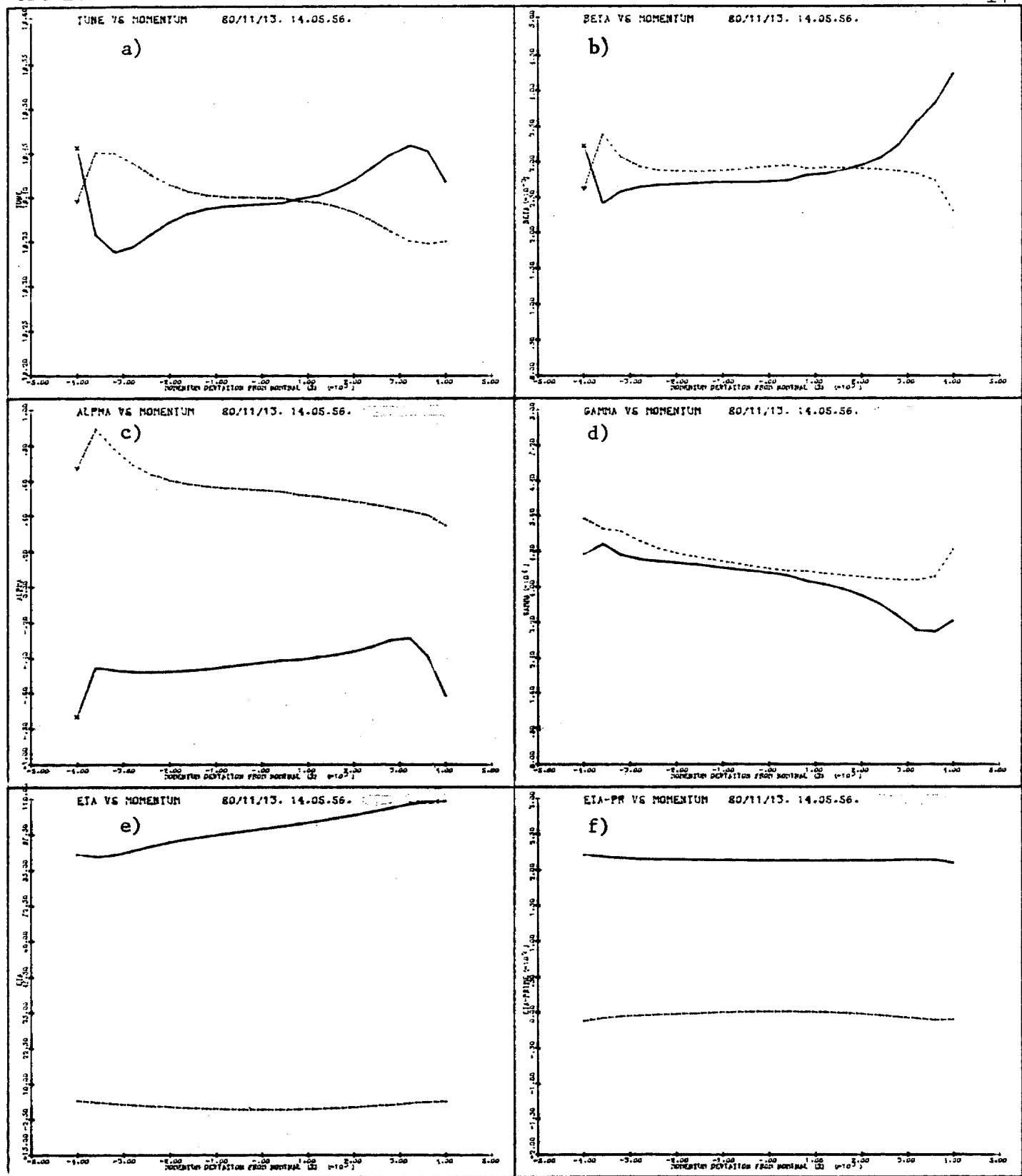


Figure 4

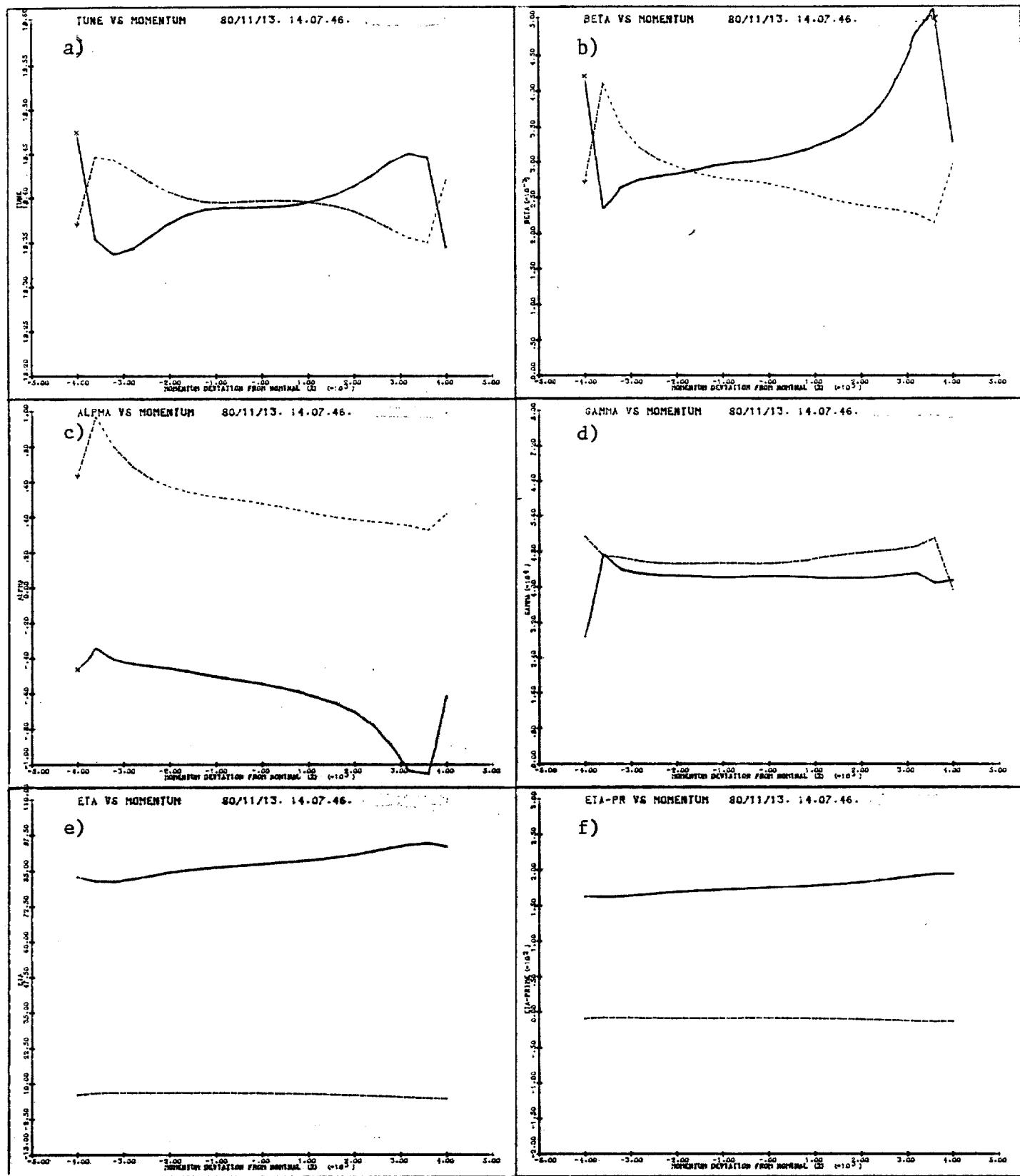


Figure 5

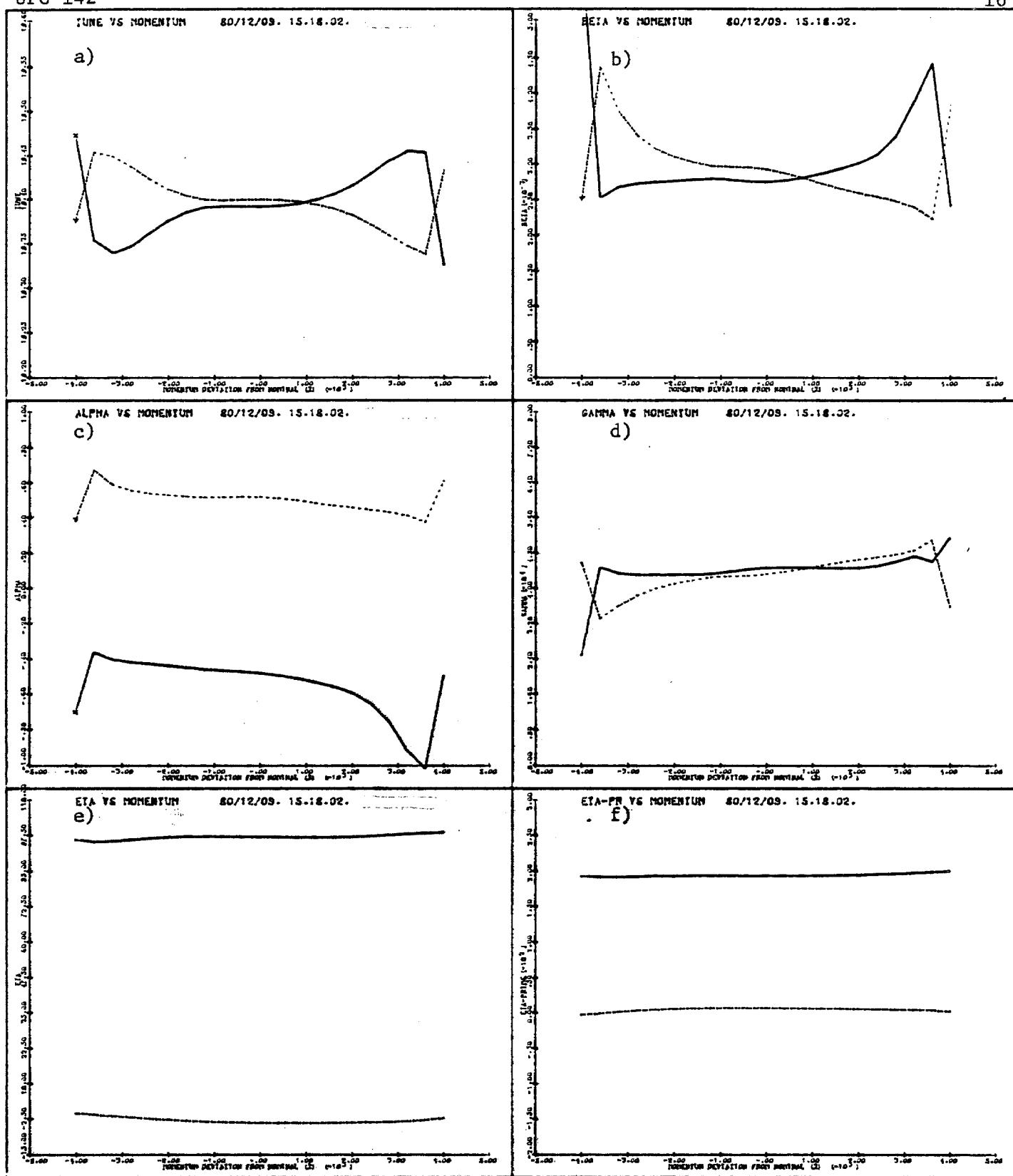


Figure 6

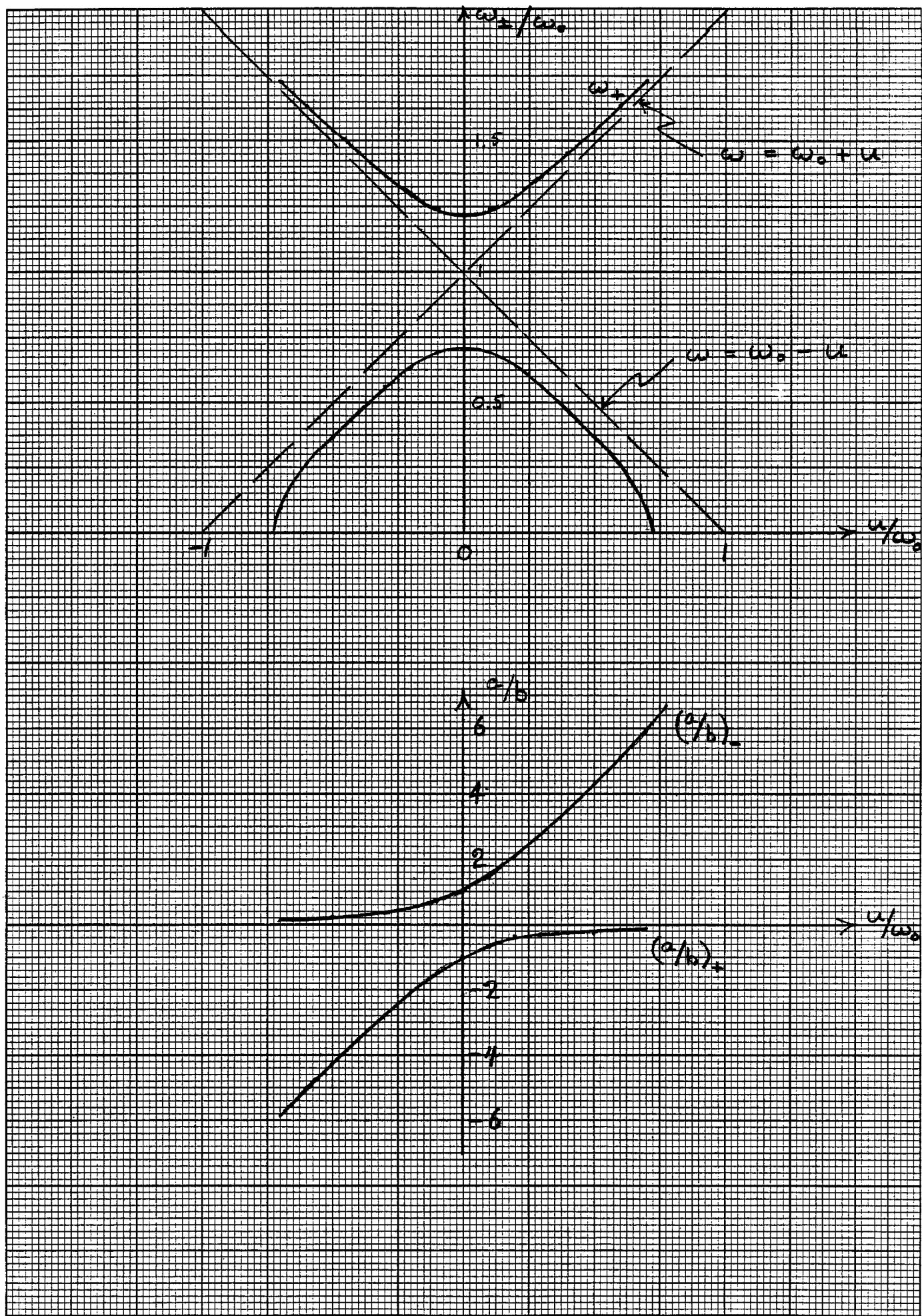


Figure 7

COS (MU) VS MOMENTUM 80/11/26. 15.45.36. FIG. 1.002

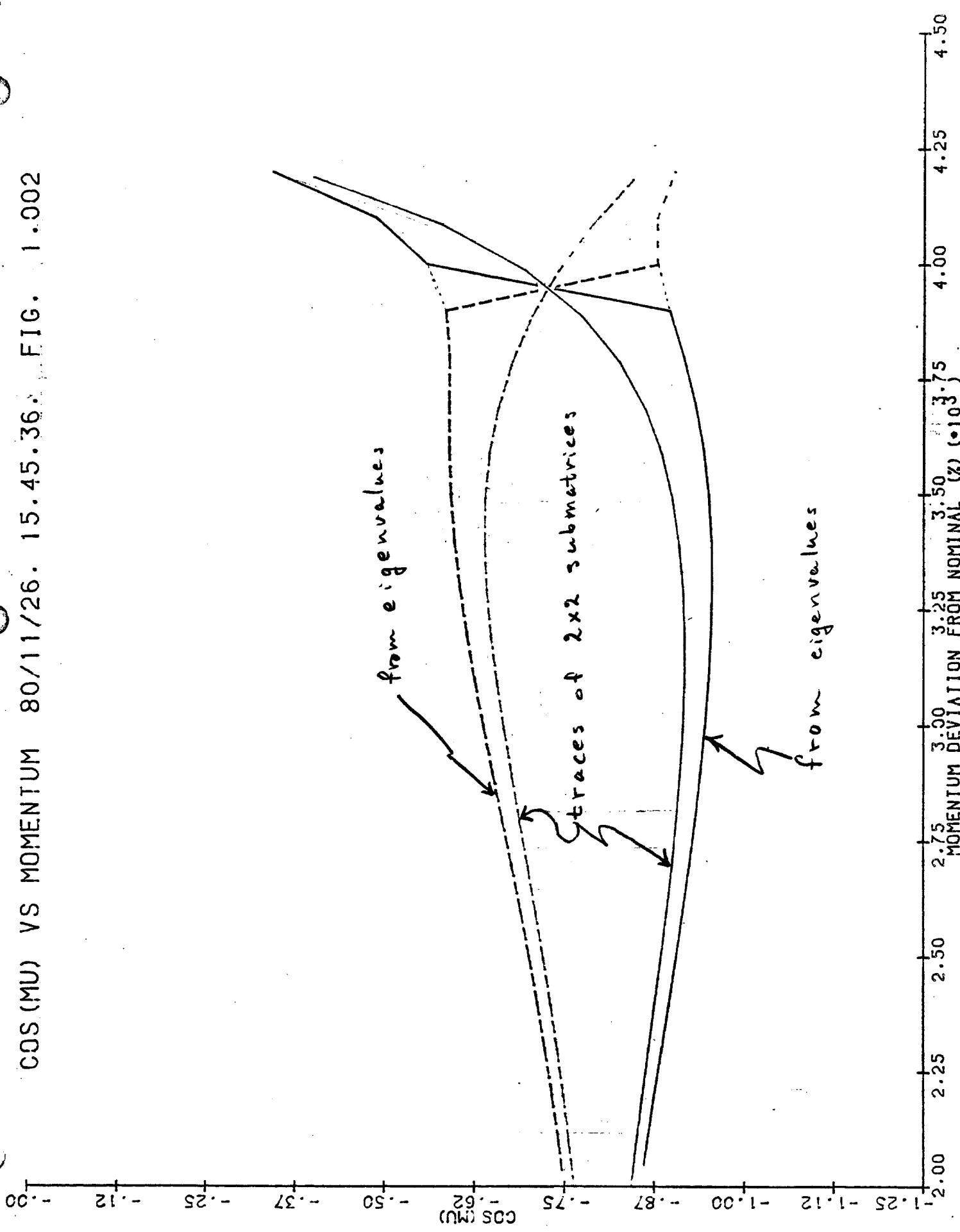


Figure 8